Semiconductor conductivity measurements using a high-sensitivity microwave technique

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An experimental study of a microwave reflection arrangement used to measure changes in the conductivity of semiconductor samples is presented. A germanium sample with associated microwave circuitry acts as a highly sensitive system whereby in the null condition almost complete absorption occurs. Changes in conductivity from the null point will cause a sharp increase in reflected microwave power. This technique will allow electrodeless measurements of both bulk and thick-film semiconductor properties and may have application as a large-area detector of infrared radiation.

INTRODUCTION

In the study of the interaction of microwaves with bulk or thick-film semiconductors, a means exists by which the electrical properties of the material may be determined. When microwave energy is transmitted through a semiconductor medium, the presence of excess free carriers can be detected by the measurement of the transmission, reflection, or absorption of the electromagnetic power. Appendix C, a glossary of the symbols used throughout the report, and Appendix D, a list of relationships, are provided as a convenience to the reader.

Other investigators have used similar microwave techniques, but with less sensitivity, to measure such properties as dielectric constant, mobility, conductivity, and lifetime. [1-11] In addition, there have been suggested device applications such as amplitude modulators, [12] and analysis directed towards a large-area infrared detector [13] has been conducted by Locke and Jacobs. Investigations have been made by Sommers et al. [14-16] on another type of reflection cavity. They investigated the behavior of a small-volume sample of semiconductor located in the high-electric-field region of a microwave reentrant cavity. An effect similar to that discussed in this report was achieved. However, their system was different from what is proposed here in that in the Sommers detector a highly directional point detector was obtained.

The purpose of this paper is to make an extended study of the reflection cavity scheme with a large area of semiconductor surface which may be used as the basis of a device for precise measurement of small changes in conductivity, or when used in the proper configuration as a sensitive acoustic and infrared detector. Major emphasis is on a semiconductor panel [germanium (Ge)] placed across a waveguide which is terminated by a sliding short. A high-sensitivity reflection cavity is formed by the tuning element and the semiconductor. By proper adjustment of the sample width and sliding short, the microwave power reflected from the face of the semiconductor can be made to approach zero. A large range of sample widths can be investigated by varying the frequency over the bandwidth of the waveguide system.

Due to the high sensitivity of the system, small changes on the order of 10^{-4} in the conductivity of the semiconductor will cause a sharp increase in the reflected microwave power linearly proportional to the total change in the conductivity of the semiconductor or as the output signal when used in a device application.

THEORY

In this section, we outline briefly the basic principles involved in the microwave system and present some of the equations describing the action of the semiconductor assembly. We shall show that this microwave semiconductor assembly can almost completely absorb or reflect the electromagnetic waves, depending upon the position of the tuning element, the semiconductor dimensions, and conductivity. The assembly or reflection cavity is made up of four layers as indicated in Fig. 1.

Medium 1 is the open waveguide through which the incident microwave energy is propagating with propagation constant Γ_1 and wave impedance Z_{01}. Here, P_{IN} represents the microwave power incident on the Ge panel, and P_R is the total reflected power. Medium 2 is the semiconductor medium with thickness l_1, conductivity σ, and permittivity ϵ. The propagation constant is γ. Medium 3 is an air space between the semiconductor material and the metallic short. This medium has an effective conductivity σ_1^1, length l, and the permittivity of free space.

In the experiment, the position of the sliding short controlled the length of this air gap. The propagation constant is Γ_3, and the wave impedance is Z_{03}. Medium 4 is the metallic sliding short.

The reflection coefficient at $X = 0$ is given by

$$\frac{E_R}{E_{in}} = \frac{Z_{(X=0)} - Z_{01}}{Z_{(X=0)} + Z_{01}}, \tag{1}$$

where E_R/E_{in} represents the reflection coefficient at the face of the semiconductor material and Z_{01} is the wave impedance in the air-filled waveguide for the TE_{10} mode. After a series of transformations of impedances, starting at the metallic short, Eq. (1) can be rewritten as follows:

$$\frac{E_R}{E_{in}} = \frac{-j\tan\beta l[(\gamma/j\beta)\tanh\gamma l_1 - 1] + [(j\beta/\gamma)\tanh\gamma l_1 - 1]}{j\tan\beta l[(\gamma/j\beta)\tanh\gamma l_1 + 1] + [(j\beta/\gamma)\tanh\gamma l_1 + 1]}. \tag{2}$$

We want a minimum reflection $E_R/E_{in} = 0$. Therefore, we set the numerator of Eq. (2) = 0, which yields

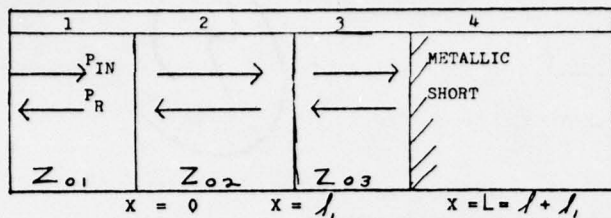$$-j\tan\beta l[(\gamma/j\beta)\tanh\gamma l_1 - 1] + [(j\beta/\gamma)\tanh\gamma l_1 - 1] = 0. \tag{3}$$

FIG. 1. Equivalent circuit for the four-media problem.

Using $\gamma = \alpha_{dg} + j\beta_{dg}$ and

$$\tanh\gamma l_1 = \frac{\tanh\alpha_{dg}l_1 + j\tan\beta_{dg}l_1}{1 + j\tanh\alpha_{dg}l_1\tan\beta_{dg}l_1},$$

we have

$$-j\tan\beta l\left[\left(\frac{\gamma}{j\beta}\right)\left(\frac{\tanh\alpha_{dg}l_1 + j\tan\beta_{dg}l_1}{1 + j\tanh\alpha_{dg}l_1\tan\beta_{dg}l_1}\right) - 1\right]$$

$$+\left[\left(\frac{j\beta}{\gamma}\right)\left(\frac{\tanh\alpha_{dg}l_1 + j\tan\beta_{dg}l_1}{1 + j\tanh\alpha_{dg}l_1\tan\beta_{dg}l_1}\right) - 1\right] = 0. \quad (4)$$

Using

$$\frac{\gamma}{j\beta} = \frac{\alpha_{dg} + j\beta_{dg}}{j\beta} = \frac{-j\alpha_{dg} + \beta_{dg}}{\beta},$$

$$\frac{j\beta}{\gamma} = \frac{j\beta}{\alpha_{dg} + j\beta_{dg}} = \frac{\beta\beta_{dg}}{\alpha_{dg}^2 + \beta_{dg}^2} + \frac{j\beta\alpha_{dg}}{\alpha_{dg}^2 + \beta_{dg}^2},$$

in Eq. (4) and clearing fractions we have, after separating into real and imaginary parts, the following:

Real

$$\left(\frac{-\alpha_{dg}}{\beta}\tan\beta l\tanh\alpha_{dg}l_1 + \frac{\beta_{dg}}{\beta}\tan\beta l\tan\beta_{dg}l_1\right.$$

$$-\tan\beta l\tanh\alpha_{dg}l_1\tan\beta_{dg}l_1$$

$$\left. + \frac{\beta\beta_{dg}}{\alpha_{dg}^2 + \beta_{dg}^2}\tanh\alpha_{dg}l_1 - \frac{\beta\beta_{dg}}{\alpha_{dg}^2 + \beta_{dg}^2}\tan\beta_{dg}l_1 - 1\right) = 0. \quad (5)$$

Imaginary

$$j\left(-\frac{\beta_{dg}}{\beta}\tan\beta l\tanh\alpha_{dg}l_1 - \frac{\alpha_{dg}}{\beta}\tan\beta l\tan\beta_{dg}l_1 + \tan\beta l\right.$$

$$+\frac{\beta\beta_{dg}}{\alpha_{dg}^2 + \beta_{dg}^2}\tan\beta_{dg}l_1 + \frac{\beta\alpha_{dg}}{\alpha_{dg}^2 + \beta_{dg}^2}\tanh\alpha_{dg}l_1$$

$$\left. -\tanh\alpha_{dg}l_1\tan\beta_{dg}l_1\right) = 0. \quad (6)$$

Assuming $\alpha_{dg} \ll \beta$ and β_{dg}, we have for Eqs. (5) and (6), after factoring, the following:

Real

$$(\beta_{dg}/\beta - \tanh\alpha_{dg}l_1)(\tan\beta l\tan\beta_{dg}l_1 - \beta/\beta_{dg}) = 0. \quad (7)$$

Imaginary

$$j[1 - (\beta_{dg}/\beta)\tanh\alpha_{dg}l_1][\tan\beta l + (\beta/\beta_{dg})\tan\beta_{dg}l_1] = 0. \quad (8)$$

Both Eqs. (7) and (8) must equal zero simultaneously in order for $E_R/E_{in} = 0$. In order for Eqs. (7) and (8) to be satisfied for minimum reflection, we must have

$$\tanh\alpha_{dg}l_1 = \beta/\beta_{dg}, \quad (9)$$

$$\tan\beta_{dg}l_1\tan\beta l = \beta/\beta_{dg}, \quad (10)$$

where α_{dg} and β_{dg} are the real and imaginary parts of the propagation constant in medium 2. We can represent α_{dg} (the attenuation constant in the semiconductor in terms of the conductivity σ and solve for l_1 in Eq. (9). Thus, we have

$$l_1 = (1/\alpha_{dg})\text{arctanh}(\beta/\beta_{dg}) \quad (11)$$

as one critical condition for zero reflection of microwave energy from the semiconductor. After making substitutions for α_{dg}, β, and β_{dg} we have

$$l_1 = (2\lambda_0/\eta_0\sigma\lambda_{dg})\text{arctanh}(\lambda_{dg}/\lambda_{ag}). \quad (12)$$

From Eq. (10) we have

$$l = (1/\beta)\arctan(\beta/\beta_{dg})1/\tan\beta_{dg}l_1. \quad (13)$$

Substituting for β and β_{dg} we have

$$l = \frac{\lambda_{ag}}{2\pi}\arctan\frac{\lambda_{dg}}{\lambda_{ag}}\frac{1}{\tan(2\pi l_1/\lambda_{dg})} \quad (14)$$

as a second condition for zero reflection. Under the assumptions made in the theory that $\alpha_{dg} < \beta_{dg}$ and β, we have thus developed the critical equations for the parameters of semiconductor thickness l_1, conductivity σ, and length of air gap l for which microwave power will be almost completely absorbed in the reflection cavity assembly. These equations indicate that by changing the microwave frequency over the bandwidth of the waveguide system, a range of sample widths may be used allowing electrodeless measurements of bulk and thick-film semiconductor properties. This theoretical analysis yielded a semiconductor thickness l_1 of 1.6×10^{-3} m at 9.0 GHz and 8.8×10^{-4} m at 7.0 GHz for Ge with $\sigma = 2.0 \ (\Omega\,\text{m})^{-1}$. However, this is an approximate analysis due to the previously mentioned assumptions. By means of the computer, the exact solution could be obtained.

EXPERIMENTAL DETAILS

All of the semiconductor samples used in conducting the experimental tests were made from an ingot of 50-Ω cm Ge. The samples were all cut into rectangular slabs approximately 5.08 cm long by 1.02 cm in height. The thickness of the samples varied according to the frequency of the incident microwave power. The sample thicknesses used in the experiments were 1.06×10^{-3} m and 4.97×10^{-4} m at 9.0 and 7.0 GHz, respectively. These sample thicknesses represent the computer-predicted optimum for approximately zero reflection from the face of the semiconductor. The surfaces of each sample were made parallel and polished so that they had a mirror finish with surface variations less than 1 μm. These samples were thoroughly cleaned and etched in CP-4. Ohmic contacts were then attached to both ends of the sample so that conductivity changes could be monitored along with microwave reflection measurements.

The samples were then fitted into a specially designed holder where the sample rested on a thin sheet of mica. As shown in Fig. 2, the sample holder was designed such that sample-retaining lugs, made of dielec-
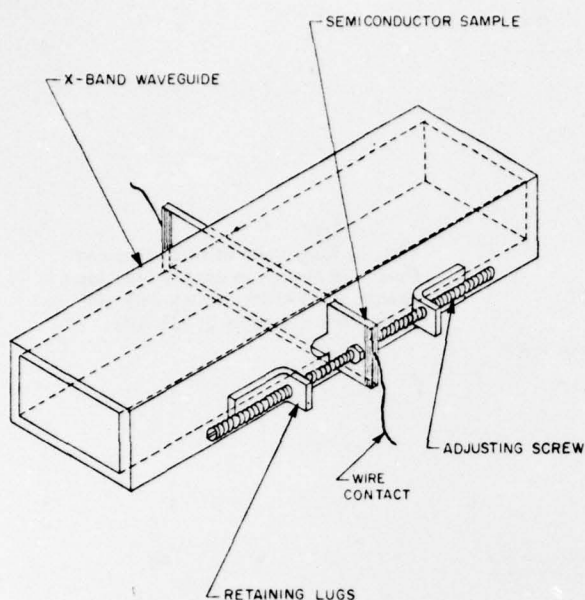
FIG. 2. Sample holder.

FIG. 3. Physical arrangement of test setup (light biasing).

tric material, on each side of the waveguide held the sample perpendicular to the propagation direction of microwave energy and snugly enough so that movement was not possible. No microwave leakage or phase shift problems have been detected from the opening in the waveguide through which the semiconductor sample was positioned in the microwave system.

The experimental setup shown in Fig. 3 was used to measure very small changes in conductivity at both 7.0 and 9.0 GHz. An ultrastable microwave oscillator was used to obtain an output signal that had a short-term frequency stability of one part in 10^8. The microwave energy propagates through two 3-dB directional couplers one of which was used to monitor the incident power level and the other to indicate the magnitude of the reflected power. By varying the position of the sliding short, the reflected power could be made to approach zero (deep null). The digital ohmmeter attached to each end of the sample measured the resistance of the semiconductor to an accuracy of four significant figures. From this measurement of the resistance of the sample, the resistivity was then calculated. The reciprocal of the resistivity yielded the sample conductivity. The spectrum analyzer, used to monitor the reflected power, had a minimum cw sensitivity of −125 dBW at a 1-kHz resolution. A monochromator was used as the infrared light source to vary the conductivity of the sample. The position of the infrared light source is shown in Fig. 4. Using this technique, small changes could be made in the conductivity of the semiconductor.

Measurements of $|E_R/E_{in}|$ were made using the following method: The ultrastable oscillator was first stabilized at the desired frequency and the output set at the desired level. The position of the sliding short was then adjusted for minimum reflected power as viewed on the face of the spectrum analyzer oscilloscope. Then, by adjusting the intensity of the infrared light
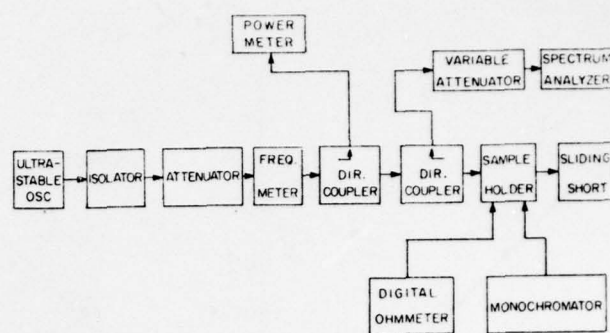
source, it was possible to bias the conductivity of the semiconductor so that the reflected microwave energy, as viewed on the oscilloscope, disappeared into the noise level of the spectrum analyzer. The reflected energy at this point is 125 dB down from the input power. The intensity of the infrared light source was then altered such that very small fluctuations in the conductivity of the sample could be monitored. A direct-reading variable attenuator in the reflection arm was then adjusted to increase the attenuation and to lower the signal amplitude to the noise level of the spectrum analyzer. In this manner, the conductivity of the sample could be varied both above and below the critical value, and thus, experimental points could be taken on either side of the reflection null.

EXPERIMENTAL RESULTS

When utilizing the present technique for monitoring very small changes in conductivity of bulk samples, a minimum detectable power level was established. This was done by placing the input or incident power directly into the spectrum analyzer. The input power was attenuated using a direct-reading attenuator until the signal disappeared into the noise with the spectrum analyzer at maximum gain. The input power was then measured, and along with the amount of attenuation used, the mini-
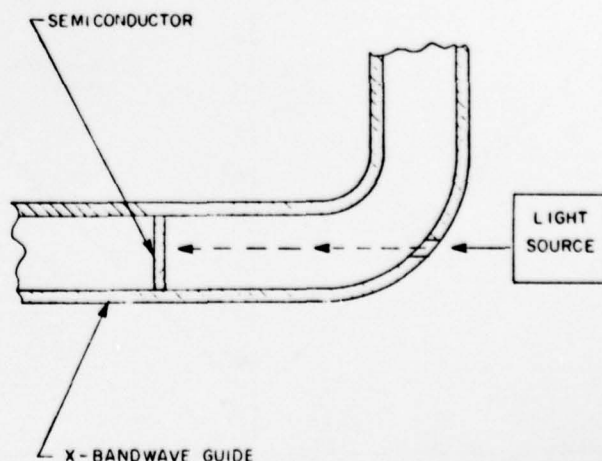

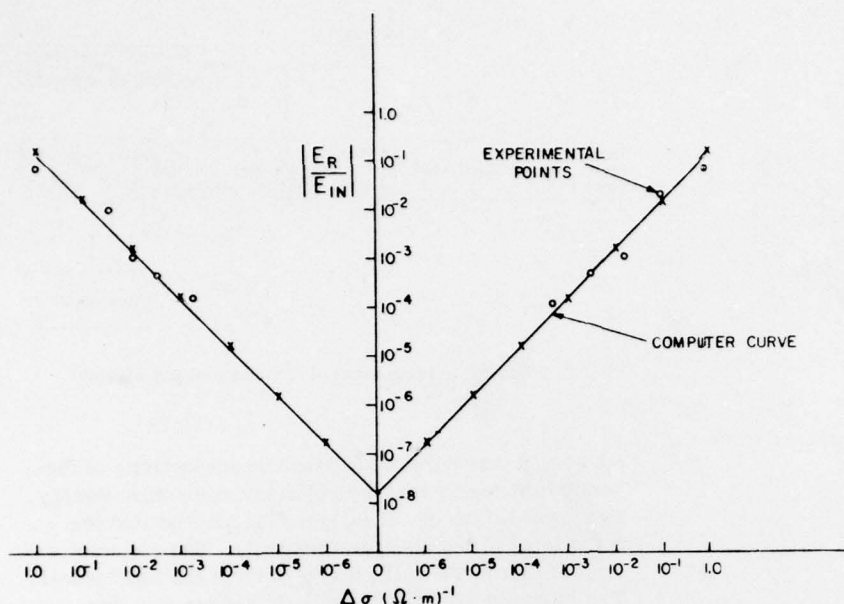FIG. 4. Position of the infrared light source.

FIG. 5. Magnitude of reflection coefficient vs change in conductivity for a sample 1.06×10^{-3} m thick with null conductivity 2.0 mho/m at 9.0 GHz.

mum detectable power was then calculated. At 7.0 and 9.0 GHz the spectrum analyzer had a minimum detectable power of 4×10^{-13} W. Figures 5 and 6 show the experimental results obtained using the sample with a thickness of 1.06×10^{-3} m and an operating frequency of 9.0 GHz. $|E_R/E_{in}|$ is plotted in Fig. 5 as a function of the change in conductivity from the null value of 2 $(\Omega\,m)^{-1}$. Figure 6 shows the results obtained when the length of medium 3 was varied from the null position. Good agreement is shown between theoretical computer predictions and the experimental data. The experimental results observed at 7.0 GHz using the 4.97×10^{-4}-m sample are shown in Figs. 7 and 8. The data obtained at 7.0 GHz using the thick-film sample compare closely to those obtained at 9.0 GHz. Here again the experimental data are in farily close agreement with the theoretical predictions.

DISCUSSION

By using a highly sensitive microwave receiving system such as a spectrum analyzer (superhetrodyne receiver), changes in conductivity of the order of 10^{-4} $(\Omega\,m)^{-1}$ producing power changes in the order of 125 dB below the incident power could be monitored. This is graphically illustrated in Figs. 5 and 7. The limitation on the accuracy with which small changes in conductivity could be measured depended upon the stability of the digital ohmmeter. Using the present technique, the smallest change in conductivity which could be measured was in the order of 10^{-4} $(\Omega\,m)^{-1}$. Figures 5 and 7 indicate that the reflection coefficients are a linear change function of small changes in conductivity, and therefore, extrapolations to smaller changes can be made from existing data. The smallest measureable
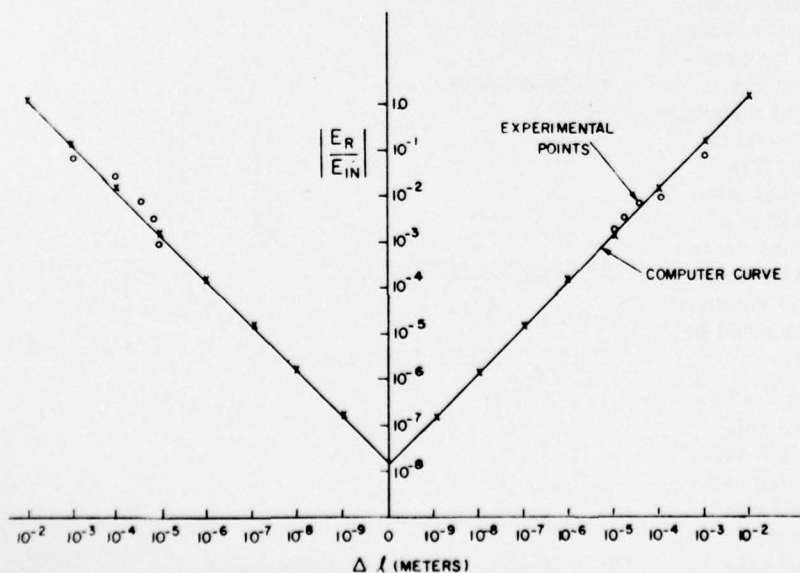

FIG. 6. Magnitude of the reflection coefficient for a sample 1.06×10^{-3} m thick at 9.0 GHz vs change in distant to short from null conditions.
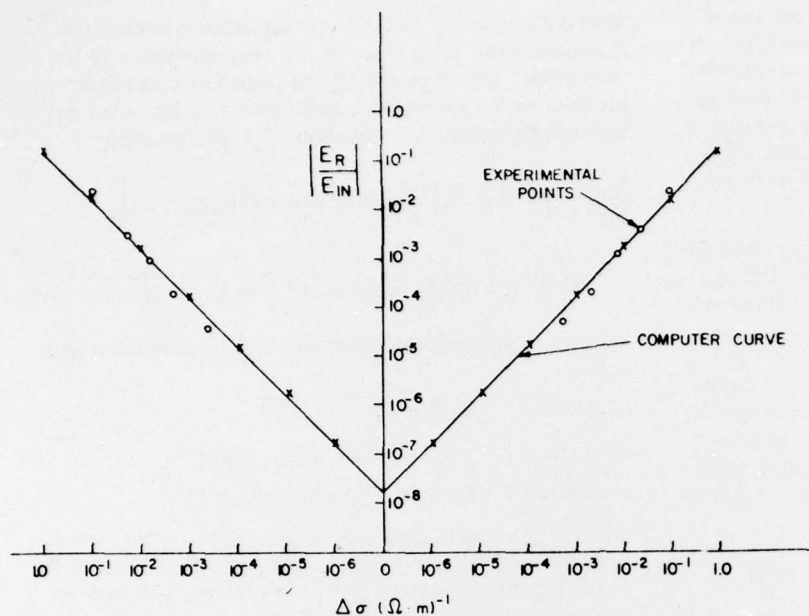
FIG. 7. Magnitude of reflection coefficient vs change in conductivity for a sample 4.97×10^{-4} m thick with null conductivity 2.0 mho/m at 7.0 GHz.

change in the short position using the present equipment was 10^{-5} m. This limitation can be improved by the use of an electronic tuning mechanism where changes in position on the order of 10^{-8} m would be measurable. However, when such small changes in the short position are made, a stable platform may be needed to eliminate vibrations. The significant thing about the data shown in Fig. 7 is that they represent measurements made on a 4.97×10^{-4}-m sample. The obvious advantage of this is that small changes in conductivity of thick-film samples can be monitored if one operates at a frequency near cutoff.

CONCLUSIONS

Experimental data have been obtained on a highly sensitive four-layer system in a X-band waveguide consisting of air, a semiconductor, an air gap with no external attenuation, and a metal reflector. The investigation

has shown that microwave power can be almost completely absorbed by the system if the conductivity of the semiconductor is adjusted and if the thickness of the semiconductor and the air gap are chosen properly. In the null condition, the microwave reflection assembly is highly sensitive to changes in the conductivity of the semiconductor and to the length of the air gap.

Using an input power of 15×10^{-6} W, the results indicate that changes in conductivity as small 10^{-4} $(\Omega\,\text{m})^{-1}$ and changes of 10^{-5} m in the short position could be measured using the present system. Operating at a frequency near cutoff allows thick-film studies to be made because at these wavelengths the critical l_1 is small. At 9.0 GHz the computer-calculated sample width was 1.06×10^{-3} m. At 7.0 GHz the sample thickness was decreased to 4.97×10^{-4} m.

The sensitivity of the present system can be increased by at least 3 orders of magnitude by increasing the input
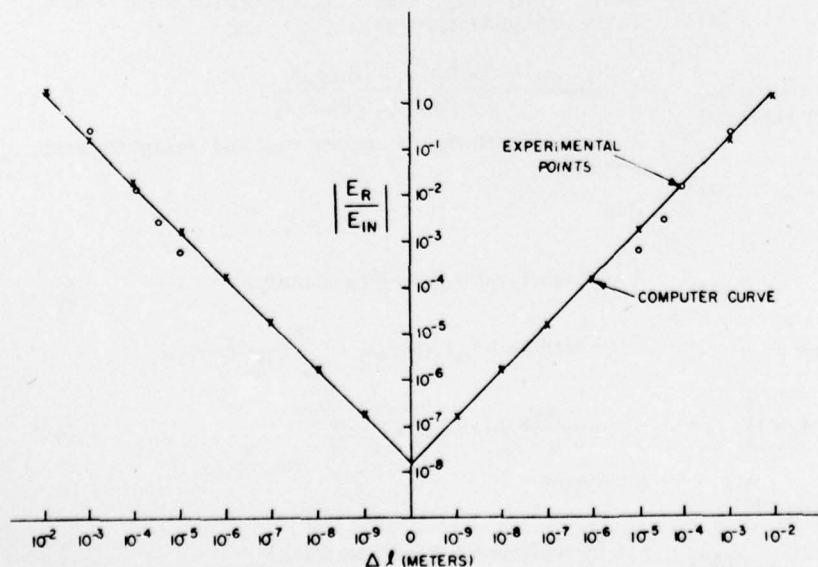

FIG. 8. Magnitude of reflection coefficient for a sample 4.97×10^{-4} m thick vs change in distance to short from null conditions at 7.0 GHz.

power level to 1 mW and by the use of a narrow-band phase-lock amplifier. This would make changes in conductivity on the order of $10^{-6}-10^{-7}$ $(\Omega \, \mathrm{m})^{-1}$ measurable. The limitation on the accuracy with which the short position could be measured can be improved by the use of an electronic tuning system which makes possible the measurement of changes on the order of 10^{-8} m in the short position.

The physical effect investigated in this report has potential application in acoustic and large-area infrared photoconductive detectors with sensitivities comparable with other state-of-the-art detectors. However, since the microwave effect utilizes a tuned reflection cavity, a high degree of stability in the frequency of the oscillator is required (at least one part in 10^6). Although all measurements were made on Ge for calibration purposes, the technique can be applied to other materials where electrodeless measurements are required due to the difficulty in making Ohmic contacts.

APPENDIX A: DERIVATION OF EQUATION FOR REFLECTION COEFFICIENT AT THE SEMICONDUCTOR SAMPLE

Assume a four-media problem as shown in Fig. 1. Medium 1 is air enclosed in a metal waveguide with propagation constant Γ_1 and wave impedance Z_{01}. Electromagnetic radiation in the dominant mode of operation is incident upon the surface of the semiconductor and is designated as E_{in}. The wave then undergoes multiple reflections in the media and reflects out the front surface. The resultant field in the reflected wave is designated E_R. Medium 2 is a semiconductor sample with conductivity σ, length l_1, and permittivity ϵ. The propagation constant is γ, and the impedance in the semiconductor is Z_{02}. Medium 3 is a hypothetical lossy medium with conductivity σ', length l_1, and the permittivity of free space. The propagation constant is Γ_3, and the impedance in the lossy medium is Z_{03}. This can be realized by an attenuator in an air waveguide followed by a movable short circuit. The reflection coefficient at $X = 0$ is given by

$$\frac{E_R}{E_{in}} = \frac{Z_{(X=0)} - Z_{01}}{Z_{(X=0)} + Z_{01}}. \tag{A1}$$

The solution may be obtained by the transformation of impedances from the metallic short. The impedance at $X = l_1$ is given by

$$Z_{(X=l_1)} = Z_{03} \tanh \Gamma_3 l. \tag{A2}$$

The impedance at $X = 0$ is given by

$$Z_{(X=0)} = Z_{02} \left(\frac{Z_{(X=l_1)} + Z_{02} \tanh \gamma l_1}{Z_{02} + Z_{(X=l_1)} \tanh \gamma l_1} \right). \tag{A3}$$

Using Eqs. (A3) and (A2) in Eq. (A1), we have

$$\frac{E_R}{E_{in}} = \left[\frac{-Z_{03}}{Z_{01}} \tanh \Gamma_3 l \left(\frac{Z_{01}}{Z_{02}} \tanh \gamma l_1 - 1 \right) + \frac{Z_{02}}{Z_{01}} \tanh \gamma l_1 - 1 \right]$$

$$\times \left[\frac{Z_{03}}{Z_{01}} \tanh \Gamma_3 l \left(\frac{Z_{01}}{Z_{02}} \tanh \gamma l_1 + 1 \right) + \frac{Z_{02}}{Z_{01}} \tanh \gamma l_1 + 1 \right]^{-1}, \tag{A4}$$

where $Z_{01} = j\omega u / \Gamma_1$ is the wave impedance in medium 1. $Z_{02} = j\omega u / \gamma$ the impedance in the semiconductor in the waveguide, and $Z_{03} = j\omega u / \Gamma_3$ the impedance in the lossy medium in the waveguide; substituting in Eq. (A4) and assuming medium 1 is lossless ($\Gamma_1 = j\beta$), we have

$$\frac{E_R}{E_{in}} = \left[\frac{-j\beta}{\Gamma_3} \tanh \Gamma_3 l \left(\frac{\gamma}{j\beta} \tanh \gamma l_1 - 1 \right) + \frac{j\beta}{\gamma} \tanh \gamma l_1 - 1 \right]$$

$$\times \left[\frac{j\beta}{\Gamma_3} \tanh \Gamma_3 l \left(\frac{\gamma}{j\beta} \tanh \gamma l_1 + 1 \right) + \frac{j\beta}{\gamma} \tanh \gamma l_1 + 1 \right]^{-1}, \tag{A5}$$

where the propagation constants for the various media are

medium 1 $j\beta = [(\pi/a)^2 - \omega^2 u \epsilon_0]^{1/2}$,

medium 2 $\gamma = [(\pi/a)^2 - \omega^2 u \epsilon_0 \epsilon_r + j\omega u \sigma]^{1/2}$,

medium 3 $\Gamma_3 = [(\pi/a)^2 - \omega^2 u \epsilon_0 + j\omega u \sigma']^{1/2}$,

and a is the width of the waveguide (m); $\omega = 2\pi f$, where f is the microwave frequency (Hz); σ is the conductivity of the semiconductor $(\Omega \, \mathrm{m})^{-1}$; σ' is the conductivity of medium 3 $(\Omega \, \mathrm{m})^{-1}$; $\epsilon_0 = 8.854 \times 10^{-12}$ f/m; ϵ_r is the relative permittivity; and $u = 4\pi \times 10^{-7}$ h/m. If medium 3 is lossless ($\sigma' = 0$), then $\Gamma_3 = j\beta$ and Eq. (A5) reduces to

$$\frac{E_R}{E_{in}} = \left[-j \tan \beta l \left(\frac{\gamma}{j\beta} \tanh \gamma l_1 - 1 \right) + \frac{j\beta}{\gamma} \tanh \gamma l_1 - 1 \right]$$

$$\times \left[j \tan \beta l \left(\frac{\gamma}{j\beta} \tanh \gamma l_1 + 1 \right) + \frac{j\beta}{\gamma} \tanh \gamma l_1 + 1 \right]^{-1}. \tag{A6}$$

For medium 3 lossless, Eq. (A6) can be used to obtain an approximate calculation for the values of l_1 and l for $|E_R/E_{in}|/\mathrm{min}$ as a function of semiconductor conductivity and frequency of wavelength. Since we want $E_R/E_{in} = 0$ in Eq. (A6), we can set the numerator equal to zero, which yields

$$-j \tan \beta l [(\gamma/j\beta) \tanh \gamma l_1 - 1] + [(j\beta/\gamma) \tanh \gamma l_1 - 1] = 0. \tag{A7}$$

Using $\gamma = \alpha_{dg} + j\beta_{dg}$, where α_{dg} is the attenuation constant in the semiconductor in the guide, and

$$\tanh \gamma l_1 = \frac{\tanh \alpha_{dg} l_1 + j \tan \beta_{dg} l_1}{1 + j \tanh \alpha_{dg} l_1 \tan \beta_{dg} l_1},$$

in Eq. (A7) and separating into real and imaginary parts we have:

Real

$$\left(\frac{-\alpha_{dg}}{\beta} \tan \beta l \tanh \alpha_{dg} l_1 + \frac{\beta_{dg}}{\beta} \tan \beta l \tan \beta_{dg} l_1 \right.$$

$$- \tan \beta l \tanh \alpha_{dg} l_1 \tan \beta_{dg} l_1 + \frac{\beta \beta_{dg}}{\alpha_{dg}^2 + \beta_{dg}^2} \tanh \alpha_{dg} l_1$$

$$\left. - \frac{\beta \alpha_{dg}}{\alpha_{dg}^2 + \beta_{dg}^2} \tan \beta_{dg} l_1 - 1 \right) = 0. \tag{A8}$$

Imaginary

$$j \left(-\frac{\beta_{dg}}{\beta} \tan \beta l \tanh \alpha_{dg} l_1 + \frac{\beta_{dg}}{\beta} \tan \beta l \tan \beta_{dg} l_1 \right.$$

$$-\tan\beta l\,\tanh\alpha_{d g}l_1\,\tan\beta_{d g}l_1+\frac{\beta\beta_{d g}}{\alpha_{d g}^2+\beta_{d g}^2}\tanh\,\alpha_{d g}l_1$$

$$-\frac{\beta\alpha_{d g}}{\alpha_{d g}^2+\beta_{d g}^2}\tan\beta_{d g}l_1-1\Big)=0. \tag{A9}$$

Real

$$(\beta_{d g}/\beta-\tanh\alpha_{d g}l_1)(\tan\beta l\,\tan\beta_{d g}l_1-\beta/\beta_{d g})=0. \tag{A10}$$

Imaginary

$$j[1-(\beta_{d g}/\beta)\tanh\alpha_{d g}l_1][\tan\beta l+(\beta/\beta_{d g})\tan\beta_{d g}l_1]=0. \tag{A11}$$

Both Eqs. (A10) and (A11) must equal zero simultaneously in order for $E_R/E_{in}=0$; therefore, there are four possible cases:

Case 1:

$$1-(\beta_{d g}/\beta)\tanh\alpha_{d g}l_1=0,$$

$$\beta_{d g}/\beta-\tanh\alpha_{d g}l_1=0.$$

Both conditions cannot be satisfied.

Case 2:

$$\tanh\alpha_{d g}l_1=\beta_{d g}/\beta,$$

$$\tan\beta l=(-\beta/\beta_{d g})\tan\beta_{d g}l,$$

where

$$\beta_{d g}=\frac{2\pi}{\lambda_{d g}}=\frac{2}{\lambda_0}\Big[\epsilon_r-\Big(\frac{\lambda_0}{\lambda_c}\Big)^2\Big]^{1/2},$$

$$\beta=\frac{2\pi}{\lambda_{a g}}=\frac{2\pi}{\lambda_0}\Big[1-\Big(\frac{\lambda_0}{\lambda_c}\Big)^2\Big]^{1/2},$$

where $\lambda_{d g}$ is the wavelength in the semiconductor in the guide, $\lambda_{a g}$ the wavelength in air in the guide, λ_0 the free-space wavelength, λ_c the cutoff wavelength of the guide, and ϵ_r the relative permittivity of the semiconductor, $\epsilon_r>1$. This requires $\tanh\alpha_{d g}l_1>1$ which is not possible.

Case 3:

$$\tan\beta l=(-\beta/\beta_{d g})\tan\beta_{d g}l_1,$$

$$\tan\beta l\,\tan\beta_{d g}l_1=\beta/\beta_{d g}.$$

This requires $(\tan\beta_{d g}l_1)^2=-1$ which is not possible.

Case 4:

$$\tanh\alpha_{d g}l_1=\beta/\beta_{d g}, \tag{A12}$$

$$\tan\beta_{d g}l_1\,\tan\beta l=\beta/\beta_{d g}. \tag{A13}$$

These are the only two conditions for minimum reflection. Solving for l_1 and l from Eqs. (A12) and (A13) yields the thickness of the semiconductor and the distance to the short for minimum reflection. From Eq. (A12), we have

$$l_1=(1/\alpha_{d g})\,\mathrm{arctanh}(\beta/\beta_{d g}), \tag{A14}$$

where

$$\beta_{d g}=2\pi/\lambda_{a g},$$

$$\beta=2\pi/\lambda_{d g},$$

$$\alpha_{d g}=\tfrac{1}{2}\eta_0\sigma\lambda_{d g}/\lambda_0,$$

where η_0 is the impedance in free space and σ is the conductivity of the semiconductor.

Substituting in Eq. (A14) yields

$$l_1=(2\lambda_0/\lambda_{d g}\sigma\eta_0)\,\mathrm{arctanh}(\lambda_{d g}/\lambda_{a g}). \tag{A15}$$

From Eq. (A13), we have

$$l=(1/\beta)\,\mathrm{arctan}(\beta/\beta_{d g})\,\tan\beta_{d g}l_1)^{-1},$$

or

$$l=(\lambda_{a g}/2\pi)\,\mathrm{arctan}(\lambda_{d g}/\lambda_{a g}[\tan(2\pi l_1/\lambda_{d g})]^{-1}. \tag{A16}$$

Knowing all parameters in Eqs. (A15) and (A16) we can compute the values of l_1 and l for minimum reflection. The reader is cautioned that this is an approximate solution because of the assumption that $\alpha_{d g}<\beta_{d g}$ and β.

APPENDIX B: EVALUATION OF D^*

This reflection cavity technique utilizing a semiconductor material has potential application as a photoconductive infrared detector. Radiation of the proper wavelength focused on the surface of the semiconductor under null conditions of the microwave circuit will generate electrons and holes and cause a change in conductivity of the sample.

The conductivity of a semiconductor sample is given by:[17]

$$\sigma=|e|(n_e\mu_e+n_h\mu_h), \tag{B1}$$

where e is the electronic charge, n_e the electron density, n_h the hole density, μ_e the electron mobility (cm²/V sec), and μ_h the hole mobility (cm²/V sec). Since in an intrinsic semiconductor $n_e=n_h=n_i$, we have

$$\sigma=|e|n_i(\mu_e+\mu_h). \tag{B2}$$

If we take an average of the mobilities as μ_i, we have

$$\sigma=|e|n_i\mu_i. \tag{B3}$$

The change in conductivity is given by

$$\Delta\sigma=|e|(\mu_i\Delta n_i+n_i\Delta\mu_i). \tag{B4}$$

Assuming that the change in the mobility term is small compared with the n_i term, we have

$$\Delta\sigma=|e|(\mu_i\Delta n_i). \tag{B5}$$

The change in the total number of carriers can be written in terms of the incident photon flux density ϕ, the carrier lifetime τ, and the effective area A of the semiconductor. Assuming that each incident photon creates an electron-hole pair (100% quantum efficiency), we have

$$\Delta n_i=\phi A\tau, \tag{B6}$$

where ϕ is the photon flux density (photons/cm² sec); A the effective area (cm²), and τ the carrier lifetime (μsec). Since the number of photons per second of energy $h\nu$ is given by

$$\phi=W/h\nu, \tag{B7}$$

where W is the radiant power (W/cm²), h is Planck's constant (W/sec²), and ν is the frequency of incident radiation (Hz), substituting for ϕ, we have

$$\Delta n_i=(W\tau/h\nu)A. \tag{B8}$$

The total change in conductivity in the sample volume can now be written as

$$\Delta\sigma = |e|AW\tau\mu_i/h\nu V, \qquad (B9)$$

or in terms of incident radiant power

$$W = \Delta\sigma h\nu l_1/e\tau\mu_i. \qquad (B10)$$

Using l_1 as 4.97×10^{-4} m and substituting in the following numerical quantities: $\tau = 100$ μsec, $A = 0.82$ cm^2 (effective area of TE$_{10}$ mode), $e = 1.6\times10^{-19}$ C, $\mu_i = 0.38$ m^2/V sec, $\lambda = 1.5$ μm, we have

$$W = \Delta\sigma(9\times10^{-4}) \text{ W}. \qquad (B11)$$

Using the present equipment, we were able to detect a change in conductivity of 10^{-4} $(\Omega\,\text{m})^{-1}$. This would give a flux intensity of 9×10^{-8} W/cm^2. The D^* of the system can be written as

$$D^* = (A\Delta f)^{1/2}/W, \qquad (B12)$$

where $A \approx 0.82$ cm^2, $\Delta f = 1000$ Hz; so that $D^* = 3.18\times10^8$.

This value of D^* can be increased by the use of a phase-lock amplifier. This would allow one to detect changes in conductivity as small as 10^{-6} $(\Omega\,\text{m})^{-1}$, or a flux intensity of 9×10^{-10} W. The subsequent change in D^* would be the following:

$$D^* = 3.18\times10^{10}.$$

This figure falls in the present state-of-the-art infrared detectors. The advantage of this technique is in the large collection angles for the incident radiation.

APPENDIX C: DEFINITION OF SYMBOLS

Medium 1 Metal waveguide, air filled

Medium 2 Metal waveguide, semiconductor filled

Medium 3 Hypothetical lossy medium in metal waveguide

Medium 4 Short-circuit termination

E_{in} Incident electric field (V/m)

E_R Total reflected field from semiconductor material (V/m)

E_R/E_{in} Reflection coefficient at face of semiconductor material

Z_{01} Wave impedance in the air waveguide for TE mode

Z_{02} Impedance in the semiconductor in the waveguide

Z_{03} Impedance in the hypothetical media in the waveguide

Γ_1 Propagation constant in medium 1, $\Gamma_1 = j\beta$ for lossless medium 1

γ Propagation constant in dielectric, $\gamma = \alpha_{d\varepsilon} + \beta_{d\varepsilon}$

Γ_3 Propagation constant in medium 1, $\Gamma_3 = \alpha_3 + j\beta_3$

$\alpha_{d\varepsilon}$ Attenuation constant in the dielectric in the guide (Np/m)

β Phase constant in air in the guide (rad/m)

λ_0 Free-space wavelength (m)

λ_d Wavelength in unbounded dielectric (m)

$\lambda_{d\varepsilon}$ Wavelength in the dielectric in the guide (m)

λ_c Cutoff wavelength of the guide (m)

V_0 Velocity of propagation; free space $V_0 = 3\times10^8$ m/sec

V_d Velocity of propagation in the dielectric (m/

η_0 Impedance of free space (377 Ω)

η Intrinsic impedance of dielectric (Ω)

ϵ_0 Free-space permittivity (8.85×10^{-12} F/m)

ϵ_r Relative permittivity

ϵ Permittivity of dielectric $(\epsilon_r\epsilon_0)$(F/m)

σ Conductivity of dielectric $(\Omega\,\text{m})^{-1}$

σ^1 Conductivity of hypothetical media $(\Omega\,\text{m})^{-1}$

l_1 Width of dielectric material (m)

l Length of short from dielectric (m)

f Frequency (Hz)

a Width of guide (m)

u_0 Permeability ($4\pi\times10^{-7}$ H/m)

APPENDIX D: LIST OF RELATIONSHIPS

$$Z_{01} = j\omega u/\Gamma_1 = \omega u/\beta$$

$$Z_{02} = j\omega u/\gamma$$

$$Z_{03} = j\omega u/\Gamma_3$$

$$\Gamma_1 = j\beta = [(\pi/a)^2 - \omega^2 u\epsilon_0]^{1/2}$$

$$\gamma = (\pi/a)^2 - \omega^2 u\epsilon_0\epsilon_r + j\omega u\sigma]^{1/2} = \alpha_{d\varepsilon} + j\beta_{d\varepsilon}$$

$$\Gamma_3 = [(\pi/a)^2 - \omega^2 u\epsilon_0 + j\omega u\sigma']^{1/2}$$

$$\alpha_{d\varepsilon} = \frac{\eta\sigma}{2[1 - (\lambda_d/\lambda_c)^2]^{1/2}} = \frac{\eta_0\sigma}{2}\frac{1}{[\epsilon_r - (\lambda_0/\lambda_c)^2]^{1/2}} = \frac{\eta_0\sigma}{2}\frac{\lambda_{d\varepsilon}}{\lambda_0}$$

$$\beta_{d\varepsilon} = \frac{2\pi}{\lambda_0}\left[\epsilon_r - \left(\frac{\lambda_0}{\lambda_c}\right)^2\right]^{1/2} = \frac{2\pi}{\lambda_{d\varepsilon}}$$

$$\lambda_{d\varepsilon} = \lambda_d\left[1 - \left(\frac{\lambda_d}{\lambda_c}\right)^2\right]^{-1/2} = \lambda_0\left[\epsilon_r - \left(\frac{\lambda_0}{\lambda_c}\right)^2\right]^{-1/2}$$
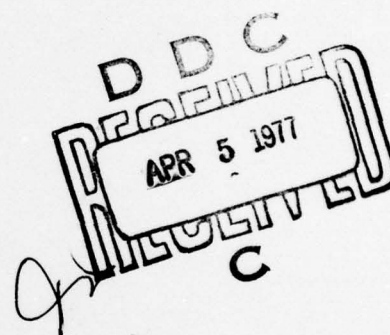
$$\lambda_{a\varepsilon} = \lambda_0\left[1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2\right]^{-1/2}$$

$$l_1 = \frac{2\lambda_0}{\lambda_{d\varepsilon}\sigma\eta_0}\text{arctanh}\frac{\lambda_{d\varepsilon}}{\lambda_{a\varepsilon}}$$

$$= \frac{2[\epsilon_r - (\lambda_0/\lambda_c)^2]^{1/2}}{\sigma\eta_0}\text{arctanh}\frac{[1 - (\lambda_0/\lambda_c)^2]^{1/2}}{[\epsilon_r - (\lambda_0/\lambda_c)^2]^{1/2}}$$

$$l = \frac{\lambda_{a\varepsilon}}{2\pi}\arctan\frac{\lambda_{d\varepsilon}}{\lambda_{a\varepsilon}}\frac{1}{\tan(2\pi l_1/\lambda_{d\varepsilon})}$$

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